

4.6 Perturbation theory

Time-independent perturbation theory

Unperturbed states	$\hat{H}_0\psi_n = E_n\psi_n$ (ψ_n nondegenerate)	(4.152)	$\hat{H}_0$ unperturbed Hamiltonian ψ_n eigenfunctions of $\hat{H}_0$ E_n eigenvalues of $\hat{H}_0$ n integer ≥ 0
Perturbed Hamiltonian	$\hat{H} = \hat{H}_0 + \hat{H}'$	(4.153)	$\hat{H}$ perturbed Hamiltonian $\hat{H}'$ perturbation ($\ll \hat{H}_0$)
Perturbed eigenvalues ^a	$E'_k = E_k + \langle \psi_k \hat{H}' \psi_k \rangle$ $+ \sum_{n \neq k} \frac{ \langle \psi_k \hat{H}' \psi_n \rangle ^2}{E_k - E_n} + \dots$	(4.154)	E'_k perturbed eigenvalue ($\simeq E_k$) $\langle \rangle$ Dirac bracket
Perturbed eigenfunctions ^b	$\psi'_k = \psi_k + \sum_{n \neq k} \frac{\langle \psi_k \hat{H}' \psi_n \rangle}{E_k - E_n} \psi_n + \dots$	(4.155)	ψ'_k perturbed eigenfunction ($\simeq \psi_k$)

^aTo second order.

^bTo first order.

Time-dependent perturbation theory

Unperturbed stationary states	$\hat{H}_0\psi_n = E_n\psi_n$	(4.156)	$\hat{H}_0$ unperturbed Hamiltonian ψ_n eigenfunctions of $\hat{H}_0$ E_n eigenvalues of $\hat{H}_0$ n integer ≥ 0
Perturbed Hamiltonian	$\hat{H}(t) = \hat{H}_0 + \hat{H}'(t)$	(4.157)	$\hat{H}$ perturbed Hamiltonian $\hat{H}'(t)$ perturbation ($\ll \hat{H}_0$) t time
Schrödinger equation	$[\hat{H}_0 + \hat{H}'(t)]\Psi(t) = i\hbar \frac{\partial \Psi(t)}{\partial t}$	(4.158)	Ψ wavefunction
	$\Psi(t=0) = \psi_0$	(4.159)	ψ_0 initial state $\hbar$ (Planck constant)/(2 π)
Perturbed wave-function ^a	$\Psi(t) = \sum_n c_n(t)\psi_n \exp(-iE_nt/\hbar)$	(4.160)	c_n probability amplitudes
	where $c_n = \frac{-i}{\hbar} \int_0^t \langle \psi_n \hat{H}'(t') \psi_0 \rangle \exp[i(E_n - E_0)t'/\hbar] dt'$	(4.161)	
Fermi's golden rule	$\Gamma_{i \rightarrow f} = \frac{2\pi}{\hbar} \langle \psi_f \hat{H}' \psi_i \rangle ^2 \rho(E_f)$	(4.162)	$\Gamma_{i \rightarrow f}$ transition probability per unit time from state i to state f $\rho(E_f)$ density of final states

^aTo first order.

